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A - L e v e l M a t h e m a t i c s

Mechanics

Sample Questions

Five extended questions with fully worked solutions
80 marks · Suggested time 1 hour 30 minutes

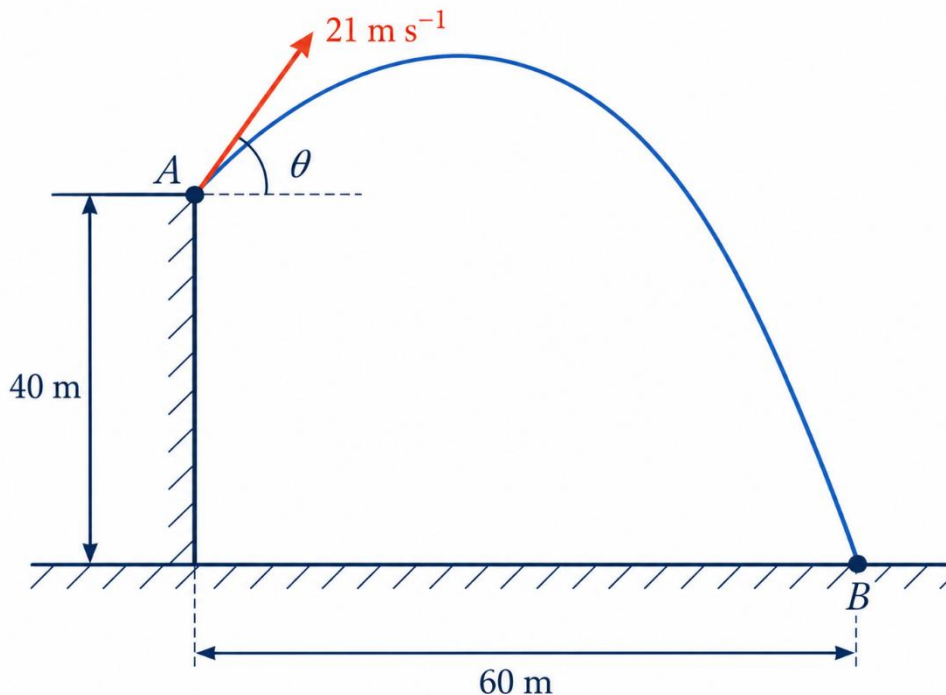
Instructions

- Answer all five questions.
- Take $g = 9.8 \text{ m s}^{-2}$ throughout.
- Give non-exact answers to three significant figures.
- Full working must be shown. Marks are awarded for method.
- Each question is followed by its worked solution on the next page.

1 *Projectile motion*

[16 marks]

A particle is projected from a point A at the top of a vertical cliff, 40 m above horizontal ground, with speed 21 m s^{-1} at an angle θ above the horizontal. The particle strikes the ground at the point B , which is 60 m horizontally from the foot of the cliff.



The particle is modelled as a projectile moving freely under gravity.

- (a) Show that $\tan \theta$ satisfies $40 \tan^2 \theta - 60 \tan \theta = 0$, and hence find θ . [5]
- (b) Find the time taken by the particle to travel from A to B. [3]
- (c) Find the speed of the particle as it strikes the ground, and the angle its velocity then makes with the horizontal. [5]
- (d) Find the greatest height above the ground reached by the particle. [3]

1 *Worked solution*

[16 marks]

(a)

Trajectory equation

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

At B: $x = 60$, $y = -40$, $u = 21$

$$-40 = 60 \tan \theta - \frac{9.8 \times 60^2}{2 \times 21^2} \sec^2 \theta$$

$$-40 = 60 \tan \theta - 40 \sec^2 \theta$$

Using $\sec^2 \theta = 1 + \tan^2 \theta$

$$-40 = 60 \tan \theta - 40 (1 + \tan^2 \theta)$$

$$40 \tan^2 \theta - 60 \tan \theta = 0$$

$$20 \tan \theta (2 \tan \theta - 3) = 0$$

$\tan \theta = 0$ rejected, since the particle is projected above the horizontal.

$$\tan \theta = \frac{3}{2} \quad \theta = 56.3^\circ$$

M1 A1 M1 M1 A1

(b)

Horizontal motion, constant speed

$$x = u \cos \theta t$$

$$60 = 21 \left(\frac{2}{\sqrt{13}} \right) t$$

$$t = 5.15 \text{ s}$$

M1 M1 A1

(c)

Components at B

$$v_x = 21 \cos \theta = 11.65$$

$$v_y = 21 \sin \theta - 9.8t = 17.47 - 50.48 = -33.0$$

Resultant

$$v = \sqrt{11.65^2 + 33.0^2} = 35.0 \text{ m s}^{-1}$$

$$\tan \alpha = \frac{33.0}{11.65} \quad \alpha = 70.6^\circ$$

B1 M1 A1 M1 A1

(d)

Vertically, at the highest point $v = 0$

$$0 = (21 \sin \theta)^2 - 2(9.8)s$$

$$s = \frac{17.47^2}{19.6} = 15.58 \text{ m}$$

$$H = 15.58 + 40 = 55.6 \text{ m}$$

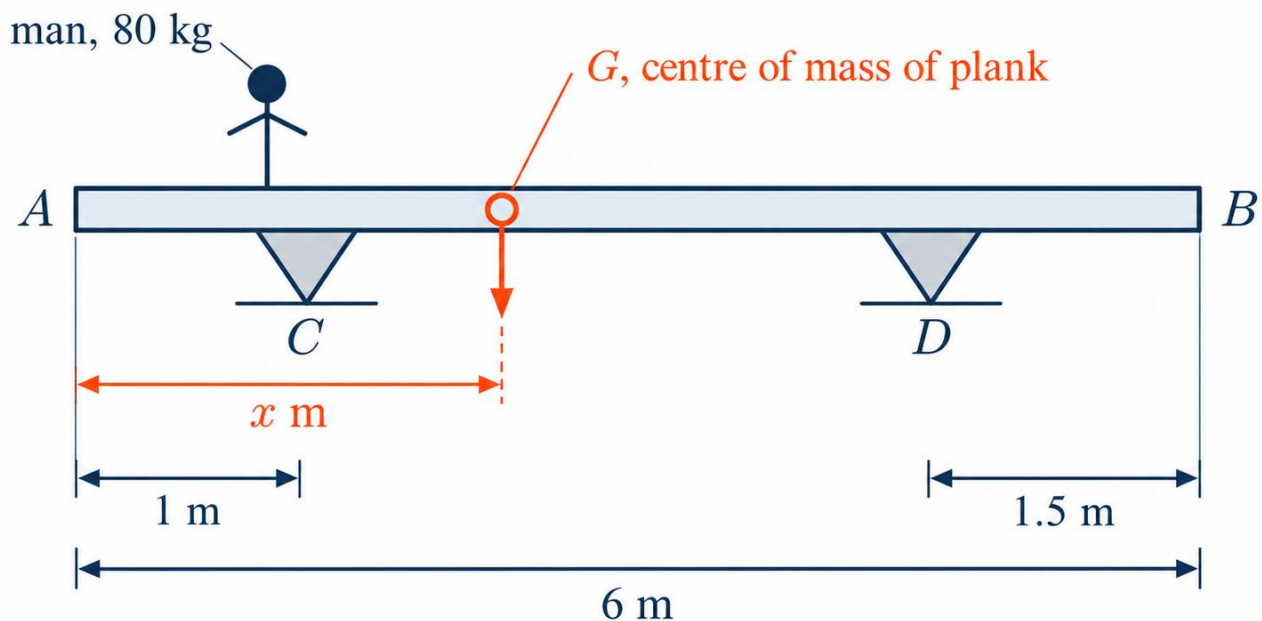
M1 A1 A1

2

Moments and tilting

[15 marks]

A non-uniform plank AB has length 6 m and mass 40 kg. It rests horizontally on two supports at C and D , where $AC = 1$ m and $DB = 1.5$ m. The centre of mass of the plank is at G , where $AG = x$ m. A man of mass 80 kg stands on the plank.



The plank is modelled as a rigid rod and the man as a particle.

- (a) When the man stands 0.5 m from A , the plank is on the point of tilting about C . Find the value of x . [5]
- (b) Find how close the man can stand to B without the plank tilting about D . [5]
- (c) The man now stands at the midpoint of CD . Find the magnitude of the reaction at each support. [5]

2

Worked solution

[15 marks]

(a)

On the point of tilting about C

$$R_D = 0$$

Moments about C

$$80g(0.5) = 40g(x - 1)$$

$$40 = 40(x - 1)$$

$$x = 2 \text{ m}$$

B1 M1 A1 M1 A1

(b)

On the point of tilting about D

$$R_C = 0$$

Moments about D, d measured beyond D

$$80gd = 40g(2.5)$$

$$d = 1.25 \text{ m}$$

$$\text{from } A : 4.5 + 1.25 = 5.75 \text{ m}$$

$$\text{distance from } B = 0.25 \text{ m}$$

B1 M1 A1 M1 A1

(c)

Man at 2.75 m from A. Moments about C, CD = 3.5 m

$$R_D (3.5) = 40g (1) + 80g (1.75) = 180g$$

$$R_D = \frac{180 \times 9.8}{3.5} = 504 \text{ N}$$

Resolving vertically

$$R_C + R_D = 120g = 1176$$

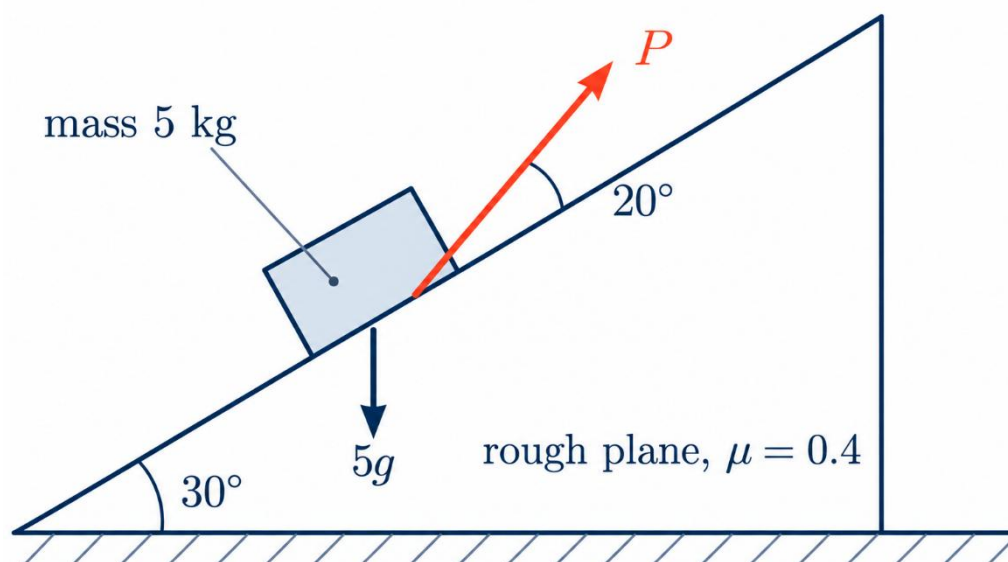
$$R_C = 1176 - 504 = 672 \text{ N}$$

M1 A1 A1 M1 A1

3 *Equilibrium on a rough plane*

[14 marks]

A block of mass 5 kg rests on a rough plane inclined at 30° to the horizontal. The coefficient of friction between the block and the plane is 0.4. A force of magnitude P newtons is applied to the block at an angle of 20° above the line of greatest slope, as shown.



The block is modelled as a particle.

- (a) Find the value of P for which the block is on the point of sliding up the plane. [6]
- (b) Find the value of P for which the block is on the point of sliding down the plane. [5]
- (c) Hence state the range of values of P for which the block remains in equilibrium, and explain briefly why a range exists. [3]

3

Worked solution

[14 marks]

(a)

Resolving perpendicular to the plane

$$R = 5g \cos 30^\circ - P \sin 20^\circ$$

On the point of sliding up, friction acts down the plane, $F = \mu R$

$$P \cos 20^\circ = 5g \sin 30^\circ + \mu R$$

$$P \cos 20^\circ = 5g \sin 30^\circ + \mu (5g \cos 30^\circ - P \sin 20^\circ)$$

$$P (\cos 20^\circ + \mu \sin 20^\circ) = 5g \sin 30^\circ + 5\mu g \cos 30^\circ$$

$$P = \frac{24.5 + 16.97}{0.9397 + 0.1368} = \frac{41.47}{1.0765}$$

$$P = 38.5 \text{ N}$$

M1 A1 M1 A1 M1 A1

(b)

Friction now acts up the plane

$$P \cos 20^\circ + \mu R = 5g \sin 30^\circ$$

$$P (\cos 20^\circ - \mu \sin 20^\circ) = 5g \sin 30^\circ - 5\mu g \cos 30^\circ$$

$$P = \frac{24.5 - 16.97}{0.9397 - 0.1368} = \frac{7.526}{0.8029}$$

$$P = 9.37 \text{ N}$$

B1 M1 A1 M1 A1

(c)

$$9.37 \text{ N} \leq P \leq 38.5 \text{ N}$$

A range exists because friction is not a fixed force. It takes whatever value up to μR is needed to maintain equilibrium, and only reaches its limiting value at the two extremes.

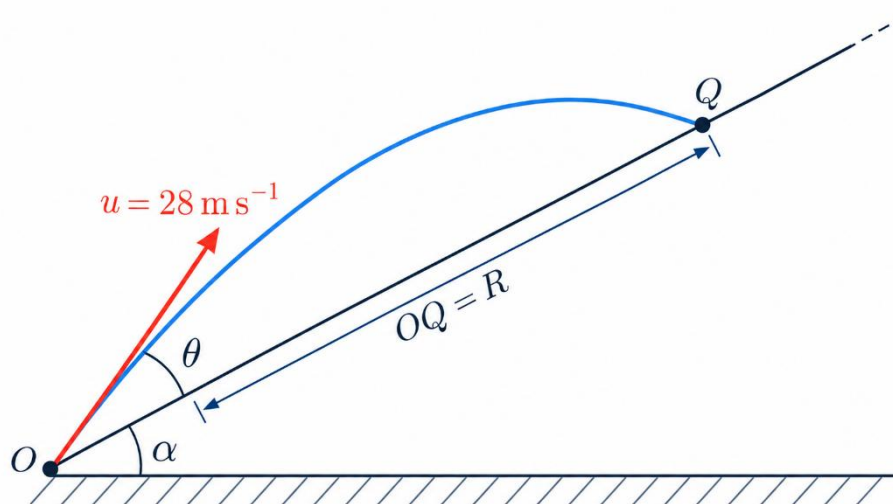
M1 A1 A1

4

Projectile on an inclined plane

[17 marks]

A particle is projected from a point O at the foot of a plane inclined at an angle α to the horizontal, with speed u at an angle θ to the inclined plane. The particle strikes the plane again at the point Q , where $OQ = R$.



In parts (b), (c) and (d) take $\alpha = 30^\circ$ and $u = 28 \text{ m s}^{-1}$.

- (a) Show that the range up the plane is given by the formula below. [7]

$$R = \frac{2u^2 \sin \theta \cos(\theta + \alpha)}{g \cos^2 \alpha}$$

- (b) Show that R is greatest when $\theta = \frac{1}{2}(90^\circ - \alpha)$, and state the value of θ . [4]
 (c) Find the maximum range up the plane. [3]
 (d) Find the time of flight corresponding to this maximum range. [3]

4

Worked solution

[17 marks]

(a)

Axes along and perpendicular to the plane

$$\text{along plane: } a = -g \sin \alpha \quad \text{perp: } a = -g \cos \alpha$$

Perpendicular: displacement zero on landing

$$0 = u \sin \theta T - \frac{1}{2} g \cos \alpha T^2$$

$$T = \frac{2u \sin \theta}{g \cos \alpha}$$

Along the plane

$$R = u \cos \theta T - \frac{1}{2} g \sin \alpha T^2$$

$$R = \frac{2u^2 \sin \theta \cos \theta}{g \cos \alpha} - \frac{2u^2 \sin^2 \theta \sin \alpha}{g \cos^2 \alpha}$$

$$R = \frac{2u^2 \sin \theta \cos(\theta + \alpha)}{g \cos^2 \alpha}$$

M1 M1 M1 A1 M1 M1 A1

(b)

Product to sum in the numerator

$$2 \sin \theta \cos(\theta + \alpha) = \sin(2\theta + \alpha) - \sin \alpha$$

$$R = \frac{u^2 [\sin(2\theta + \alpha) - \sin \alpha]}{g \cos^2 \alpha}$$

$$\text{max when } \sin(2\theta + \alpha) = 1 \Rightarrow 2\theta + \alpha = 90^\circ$$

$$\theta = \frac{1}{2} (90^\circ - \alpha) = 30^\circ$$

*M1 A1 M1 A1***(c)**

$$R_{\max} = \frac{u^2 (1 - \sin \alpha)}{g \cos^2 \alpha} = \frac{u^2}{g (1 + \sin \alpha)}$$

$$R_{\max} = \frac{28^2}{9.8 \times 1.5} = 53.3 \text{ m}$$

*M1 A1 A1***(d)**

$$T = \frac{2 (28) \sin 30^\circ}{9.8 \cos 30^\circ}$$

$$T = 3.30 \text{ s}$$

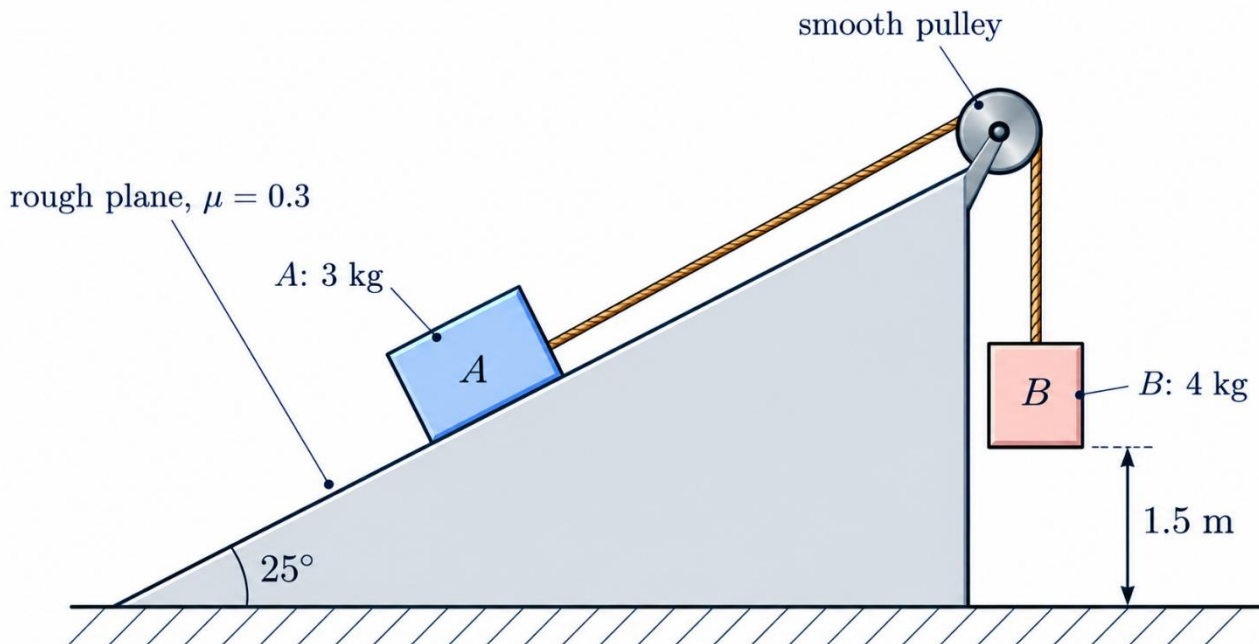
M1 M1 A1

5

Connected particles

[18 marks]

A particle A of mass 3 kg lies on a rough plane inclined at 25° to the horizontal. The coefficient of friction between A and the plane is 0.3 . A light inextensible string attached to A passes over a smooth pulley fixed at the top of the plane and carries a particle B of mass 4 kg hanging freely. The portion of the string from A to the pulley is parallel to the line of greatest slope. B is 1.5 m above horizontal ground, and the system is released from rest.



- (a) Show that the system does move when released. [2]
- (b) Find the acceleration of the system and the tension in the string. [7]
- (c) B descends to the ground and does not rebound. Find the speed of B as it reaches the ground. [3]
- (d) Find the further distance travelled by A up the plane after B has landed. [4]
- (e) Show that A then slides back down the plane. [2]

5

Worked solution

[18 marks]

(a)

Driving force

$$4g = 39.2 \text{ N}$$

Total resistance on A

$$3g \sin 25^\circ + \mu (3g \cos 25^\circ) = 12.43 + 7.99 = 20.4 \text{ N}$$

$$39.2 > 20.4 \quad \therefore \text{the system moves}$$

M1 A1

(b)

Equation of motion for each particle

$$B : 4g - T = 4a$$

$$A : T - 3g \sin 25^\circ - \mu (3g \cos 25^\circ) = 3a$$

Adding to eliminate T

$$4g - 3g \sin 25^\circ - 3\mu g \cos 25^\circ = 7a$$

$$a = \frac{39.2 - 20.42}{7} = \frac{18.78}{7}$$

$$a = 2.68 \text{ m s}^{-2}$$

$$T = 4(9.8 - 2.683) = 28.5 \text{ N}$$

M1 A1 M1 A1 A1 M1 A1

(c)

Uniform acceleration through 1.5 m from rest

$$v^2 = 0 + 2(2.683)(1.5) = 8.049$$

$$v = 2.84 \text{ m s}^{-1}$$

M1 M1 A1

(d)

String slack, so A decelerates under gravity and friction

$$T = 0 \quad \text{deceleration} = g \sin 25^\circ + \mu g \cos 25^\circ = 6.806$$

$$0 = 8.049 - 2(6.806)s$$

$$s = 0.591 \text{ m}$$

B1 M1 M1 A1

(e)

At the highest point, compare weight component with limiting friction

$$g \sin 25^\circ = 4.14 \quad \mu g \cos 25^\circ = 2.66$$

$$4.14 > 2.66 \quad \therefore A \text{ slides back down}$$

M1 A1

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